|  |  |
| --- | --- |
| optimal value : | 1. for all ; |
|  | 2. exists a sequence such that as |

unbounded problem: exists a sequence such that

infeasible problem: feasible solution

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Linear function : | | | | | | | | | | | | | | | 1. for all and for all | | | | | | | | |
|  | | | | | | | | | | | | | | | 2. for all and for all | | | | | | | | |
|  | | | | | | | | | | | | | | | , such that | | | | | | | | |
|  | | | | | | | | | | | | | | | Both convex and concave | | | | | | | | |
| generic (P): | | | | | | |  | | | | | | | | | | | | | | | | |
| convex (P): | | | | | | | 1. is convex; 2. Feasible region S is convex: concave, convex, linear | | | | | | | | | | | | | | | | |
| linear (P): | | | | | | |  | | | | | | | | | | | | | | | | |
| convex set : | | | | | | | | | we have , otherwise nonconvex. | | | | | | | | | | | | | | |
| convex function : | | | | | | | | | , we have . | | | | | | | | | | | | | | |
| concave function : | | | | | | | | | , we have . | | | | | | | | | | | | | | |
|  | | | | | | | | | is concave is convex | | | | | | | | | | | | | | |
| Epigraph of : | | | | | | | | |  | | | | | | | | | | | | | | |
|  | | | | | | | | | is a convex function is a convex set. | | | | | | | | | | | | | | |
| Level Set: | | | | | | | | | ; | | | | | | | | | | | (so ) | | | |
| Hyperplane: | | | | | | | | | , where | | | | | | | | | | | | | | |
|  | | | | | | | | | every hyperplane is a convex set | | | | | | | | | | | i.e. if is linear, is convex | | | |
|  | | | | | | | | |  | | | | | | | | | | | i.e. | | | |
| sublevel set of : | | | | | | | | |  | | | | | | | | | | |  | | | |
| superlevel set of : | | | | | | | | |  | | | | | | | | | | |  | | | |
|  | | | | | | | | | is convex set | | | | | | | | | | | | | | |
|  | | | | | | | | | is convex set | | | | | | | | | | | | | | |
| halfspace | | | | | | | | | , where | | | | | | | | | | | | | | |
| halfspace | | | | | | | | | , where | | | | | | | | | | | | | | |
|  | | | | | | | | | Linear function is a both convex and a concave function | | | | | | | | | | | | Every halfspace is a convex set. | | |
| convex (P) property | | | | | | | | | feasible region and the set of optimal solutions is a convex set. | | | | | | | | | | | | | | |
| vertex : | | | | | | | | | is convex set, ; 2. hyperplane such that  3. (i) ; (ii) | | | | | | | | | | | | | | |
|  | | | | | | | | | such a hyperplane is called a supporting hyperplane of . | | | | | | | | | | | | | | |
|  | | | | | | | | | is convex set, is vertex of C linear fun such that is unique solution of (P) min s.t. | | | | | | | | | | | | | | |
| polyhedral | | | | | | | | |  | | | | | | | | | | | | | | |
|  | | | | | | | | | is convex | | | | | | | | | | | | | | |
|  | | | | | | | | | Feasible region of linear (P) is polyhedral | | | | | | | | | | | | | | |
| bounded set | | | | | | | | | is bounded: such that all | | | | | | | | | | | | | | |
| ploytope | | | | | | | | | A *bounded polyhedron* is called a polytope | | | | | | | | | | | | | | |
| Polyhedron | | | | | | | | | | | | | | | | | | | | | | | |
| Indices of active constraint at | | | | | | | |  | | | | | | | | | | | | | | | |
| Basic solution | | | | | | | 1. all the equality constraints are active at (i.e., )  2. contains linearly independent vectors (i.e. spans ). | | | | | | | | | | | | | | | | |
| Basic feasible solution | | | | | | | | | | 1. basic solution; 2. feasible solution | | | | | | | | | | | | | |
| 一一对应 | | | | | | | P is polyhedron, , is basic feasible solution is a vertex of | | | | | | | | | | | | | | | | |
| Polyhedron contains a line | | | | | | | | | exists a vector and a nonzero vector such that for every real number λ | | | | | | | | | | | | | | |
|  | | | | | | | P is nonempty polyhedron | | | | | | | | | | | P has at least one vertex it does not contain a line (bounded) | | | | | |
|  | | | | | | |  | | | | | | | | | | | P contains at most a finite number of vertices. | | | | | |
|  | | | | | | | is a nonempty polytope. Then, has at least one vertex. | | | | | | | | | | | | | | | | |
| is a nonempty polyhedron that contains at least one vertex. For the set of optimal solutions , is nonempty. Then, contains at least one vertex of P | | | | | | | | | | | | | | | | | | | | | | | |
| Polyhedron in standard form | | | | | | | |  | | | | | | | | | | | | | | | |
|  | | | | | | | | P is a nonempty polyhedron in standard form. Then P has at least one vertex. | | | | | | | | | | | | | | | |
| ; i.e. all column | | | | | | | | | | | | | | | | | | | | | | | |
| For , . | | | | | | | | | | | | | | is a basic solution of has full column rank | | | | | | | | | |
| + full rank | | | | | | | | | | | | | | is a basic solution of and | | | | | | | | | |
| If standard linear (P) has a finite optimal value, then the set of optimal solutions , is nonempty.  Furthermore, contains at least one vertex of P. | | | | | | | | | | | | | | | | | | | | | | | |
| Every linear programming problem with a finite optimal value has at least one optimal | | | | | | | | | | | | | | | | | | | | | | | |
| Fundamental Theorem of Linear Programming | | | | | | | | | | | | | | (P) denote a linear programming problem in standard form: | | | | | | | | | |
|  | | | | | | | | | | | | | 1. (P) is infeasible and | | | | | | | | | | |
|  | | | | | | | | | | | | | 2. The optimal value is finite and there is at least one vertex in the set of optimal solutions(i.e., ) | | | | | | | | | | |
|  | | | | | | | | | | | | | 3. (P) is unbounded and | | | | | | | | | | |
| feasible direction at | | | | | | | | | | | | | | | ,exists such that | | | | | | | | |
|  | | | | | | | | . is an optimal solution of (P) for all feasible directions at . | | | | | | | | | | | | | | | |
|  | set of all feasible directions at | | | | | | | | | | | | | | | | | | | | | | |
|  |  | | | | | | | | | | | | | | If ˆx is nondegenerate, then . | | | | | | | | |
| Let   be a vertex of . If for each , then is an optimal solution of (P). | | | | | | | | | | | | | | | | | | | | | | | |
| reduced cost of the variable | | | | | | | | | | | | | | | () | | | | | | | | |
| degeneracy | | | | | | | A basic solution of P is said to be degenerate if .  Otherwise (i.e., if ), it is said to be nondegenerate. | | | | | | | | | | | | | | | | |
|  | |  | | | | | | | | | | | | | | | | means all basic variables not zero nondegenerate | | | | | |
| Standard linear (P), is a vertex of P, for each is an optimal solution of (P). | | | | | | | | | | | | | | | | | | | | | | | |
| Standard linear (P), is a nondegenerate vertex of P, is an optimal solution of (P)  for each . | | | | | | | | | | | | | | | | | | | | | | | |
| i.e. 退化vertex在有时也有可能是opt，此时看优化方向 , 如果 is feasible, 则不是最优 | | | | | | | | | | | | | | | | | | | | | | | |
|  | | | | | | | | | | | | | |  | | | | | | | | | |
|  | | | is a nonempty polyhedron | | | | | | | | | | | | | | | | | | | | |
|  | | | is a nonempty polyhedron a solution such that . | | | | | | | | | | | | | | | | | | | | |
|  | | | if A has full row rank and is a vertex of such that , then is a vertex of P | | | | | | | | | | | | | | | | | | | | |
| Two-Phase Method | | | | | | | | | (try to find vertex of (P)) | | | | | | | | | | | | | | |
|  | | | | | | | | | If opt value=0 of () in is a vertex of (P) | | | | | | | | | | | | | | |
|  | | | | | | | | | In dictionary, should be replaced by | | | | | | | | | | | | | | |
|  | | | | | | | | | Phase1 最后，将等号左侧全部换成, 然后把所有去掉变成Phase2 | | | | | | | | | | | | | | |
| a linear (P) with variables : the simplex method with the most reduced cost performs iterations | | | | | | | | | | | | | | | | | | | | | | | |
| Relaxation | | | | | | | ; choose and | | | | | | | | | | | | | | | | |
|  | | | | | | | relaxation of (P): ; so we have | | | | | | | | | | | | | | | | |
| Duality Theory | | | | | | | |  | | | | | | | |  | | | | | | | |
|  | | | | | | | | is relaxation of ; if , then , 需要 | | | | | | | | | | | | | | | |
|  | | | | | | | | ; for primal (P) and dual (D): 是下界 | | | | | | | | | | | | | | | |
|  | | | | | | | | Primal-Dual Symmetry: the dual of the dual is the primal | | | | | | | | | | | | | | | |
|  | | | | | | | | | | | | | | | Dual Problem | | | | | | | | |
|  | | | | | | | | | | | | | | | Finite optimal value | | | | | Unbounded | | | Infeasible |
| Primal (P) | | | | | Finite optimal value | | | | | | | | | | 1 | | | | | 0 | | | 0 |
|  | | | | | Unbounded | | | | | | | | | | 0 | | | | | 0 | | | 1 |
|  | | | | | Infeasible | | | | | | | | | | 0 | | | | | 1 | | | 1 |
| Complementary Slackness | | | | | | | | is feasible solutions of (P) and (D),  is optimal solutions | | | | | | | | | | |  | | | | |
| If is an optimal solution of (P), is optimal solution of (D) | | | | | | | | | | | | | | | | | | | | | | | |
| Each dictionary, is a basic feasible solution, is basic but unsure feasible | | | | | | | | | | | | | | | | | | | | | | | |
|  | | | | 1. is feasible: is optional | | | | | | | | | | | | | 2.  is not: continue | | | | | | |
|  | | | | Iterations work towards dual feasibility (对(P)计算，朝着feasible方向走) | | | | | | | | | | | | | | | | | | | |
| Sensitivity Analysis | | | | | | | | |  | | | | | | | | | | | Dual | |  | |
|  | | | | | | remains optimal | | | | | | | | | |  | | | | | | | |
|  | | | | | |  | | | | | | | | | | basic variables: ; nonbasic variables: | | | | | | | |
|  | | | | | | primal infeasible | | | | | | | | | | dual simplex method | | | | | | | |
|  | | | | | |  | | | remain optimal | | |  | | | | | | | | | | | |
|  | | | | | |  | | | Not optimal | | | primal simplex method | | | | | | | | | | | |
|  | | | | | |  | | | remain optimal | | | , , where | | | | | | | | | | | |
|  | | | | | |  | | |  | | | primal sol: ; obj fun value: | | | | | | | | | | | |
|  | | | | | |  | | | Not optimal | | | primal simplex method | | | | | | | | | | | |
|  | | | | | |  | | | remain optimal | | | , 变化等价于 | | | | | | | | | | | |
|  | | | | | |  | | | Not optimal | | | primal simplex method | | | | | | | | | | | |
|  | | | | | |  | | |  | | | Two-Phase Method from start | | | | | | | | | | | |
| add | | | | | | basic feasible solution of the new: | | | | | | | | | | | | | | | | | |
|  | | | | | | coefficients of of Row1 using ; primal simplex method | | | | | | | | | | | | | | | | | |
| Add a New | | | | | | | | | | | If satisfies , still opt | | | | | | | | | | | | |
|  | | | | | | | | | | | Otherwise, Two-Phase Method from start | | | | | | | | | | | | |
| Add a New | | | | | | | | | | | If satisfies , still opt | | | | | | | | | | | | |
|  | | | | | | | | | | | Otherwise | | | | | Add a nonnegative such that | | | | | | | |
|  | | | | | | | | | | |  | | | | |  | | | | | | | |
|  | | | | | | | | | | |  | | | | | is a basic solution of the modified primal problem | | | | | | | |
|  | | | | | | | | | | |  | | | | | dual simplex method | | | | | | | |
|  | | | | | | | | | | | | | | | | | | | | | | | |